# TutorTube: General Probability Concepts

Fall 2020

## Introduction

Hello, welcome to another edition of TutorTube, where the Learning Center's Lead Tutors help you understand challenging course concepts with easy to understand videos. My name is Kelly Schmidt, Lead Tutor for statistics at the Learning Center. In today's video, we will explore some of the basic probability concepts that will help you to be successful in your statistics courses. Specifically, we'll look at probability rules and conditional probability. Let's get started!

## General Addition Rule ("OR")

When we talk about probability, we are often concerned with the likelihood of a certain event, or outcome, occurring. For example, if we were playing cards, I might be concerned with the probability of drawing an ace, or the probability of drawing a spade. Each of these scenarios (drawing an ace or drawing a spade) is called an **event.** In order to save time, we will often use the notation $P(Ace)$ or $P(Spade)$ to refer to the probability of events like this.



**Sample Space for Choosing a Card from a Deck**

*Source: (Poston, 2017)*

Ok, so how do we actually calculate the probability of an event? Let's use $P(Ace)$ as an example. We know there are 52 cards in a standard deck. Of those 52 cards, 4 are aces: an ace of spades, ace of diamonds, ace of hearts, and ace of clubs. To find the probability of drawing any one of these four aces, we simply need to find the value of $\frac{4}{52}$, which comes out to 0.0769 or about a 7.7% chance.

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However, what if I asked, "What is the probability of drawing a card that is an ace **OR** a spade?" (Which would be written as $P(Ace\ OR\ Spade)$ in mathematical notation). This question isn't as straightforward to answer. Luckily, we have a formula that will help us.

The **general addition rule** says that if you have two events, A and B:

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

We usually use the general addition rule when we see the word "OR" in a problem: when asked to find the probability of event A **OR** event B occurring.

## Practice #1

Let's do a practice problem to see how this works.

*A probability experiment is conducted in which the sample space of the experiment is*

$$S = \{a, b, c, d, e, f, g, h, j, k, l\},$$

$$event\ F = \{d, e, f, g, h\},$$

$$and\ event\ G = \{h, j, k, l\}.$$

*Assume each outcome is equally likely. Find $P(F\ or\ G)$.*

First off, notice that the question asked us for F **or** G. This is a cue that we will need the General Addition Rule. First, we start by finding $P(F)$. To get a concrete idea of what we are doing, think of our sample space, S, as a bag filled with the letters $a$ through $l$. $P(F)$ is the probability of randomly picking a letter out of the bag and getting one of the letters that are in set F, either $d, e, f, g$, or $h$.

So:

$$P(F) = \frac{\#\ of\ things\ in\ F}{\#\ of\ things\ in\ S} = \frac{5}{12}$$

Similarly, we also find $P(G)$.

$$P(G) = \frac{\#\ of\ things\ in\ G}{\#\ of\ things\ in\ S} = \frac{4}{12}$$

Next, we need to find the number of items that are in both F and G. The only letter that is common to both is $h$.

$$F\ and\ G = \{the\ things\ that\ are\ in\ both\ F\ and\ G\} = \{h\}$$

Almost there, now we find $P(F \text{ and } G)$. Remember, we can think of this as the probability of randomly drawing the letter $h$ out of the sample space bag.

$$P(F \text{ and } G) = \frac{\# \text{ of things in } F \text{ and } G}{\# \text{ of things in } S} = \frac{1}{12}$$

In the last step, we use the General Addition Rule and plug in the values that we found.

$$P(F \text{ or } G) = P(F) + P(G) - P(F \text{ and } G)$$

$$= \frac{5}{12} + \frac{4}{12} - \frac{1}{12} = \frac{8}{12}$$

$$= .667$$

## Multiplication Rule ("AND")

In addition to the General Addition Rule, we also have the Multiplication Rule for Independent Events and the "At-Least" or Complement Rule.

These rules are similar in that we will use them when we encounter a problem asking us to find the probability of event A **AND** event B occurring.

The multiplication rule says that if we have two (or more) independent events, we can find the probability of both of them occurring by simply multiplying their respective probabilities together. Written out, the **Multiplication Rule** for two independent events reads as:

$$P(A \text{ and } B) = P(A) * P(B)$$

Additionally, we also have **the Complement ("At Least") Rule**. The formula for this rule is:

$$P(at \text{ } least \text{ } A) = 1 - P(A^c)$$

Essentially, we use this rule if we ever see the phrase "at least" in a problem. Let's look at an example to see how these rules can be used in a typical problem.

## Practice #2

*The probability that a randomly selected cat will live to age 12 is 0.63.*

    1. *Find the probability that 2 randomly selected cats will live to age 12.*

Okay, the problem tells us that the probability of a single random cat living to old age is .63 or 63%. We can represent this as $P(Cat) = .63$. Our scenario for the problem asks for the probability of encountering two random cats, lets call them Cat 1 and Cat 2, that both live to age 12. We can write this as

$$P(Cat_1 \textbf{ and } Cat_2)$$

Notice that we are dealing with an "and" problem instead of an "or" problem this time. Also, since we are assuming that these cats are random and don't have any influence on each other, we can consider each cat to be independent. Because of this, we know that we can use the Multiplication Rule for independent events.

$$P(Cat_1 \textbf{ and } Cat_2) = P(Cat_1) * P(Cat_2)$$

This rule tells us that the probability of both cats living past age twelve can be found by simply multiplying the probability that Cat 1 lives ($P(Cat_1)$) by the probability that Cat 2 lives ($P(Cat_2)$).

Since Cat 1 and Cat 2 are both random cats, we know that each of these probabilities is the same (.63). So plugging this into our formula, we get:

$$P(Cat_1 \textbf{ and } Cat_2) = (.63)(.63)$$

$$= (.63)^2$$

$$= 0.37$$

So, there is a .37 probability or a 37% of selecting two random cats that both live past age twelve.

Ok, now let's look at the next part of the problem:

### 2. Find the probability that 4 randomly selected cats will live to age 12.

This problem is similar to the last one, except now we have four cats. The cats are still independent, so the multiplication rule still applies; we can still just multiply their independent probabilities.

$$P(Cat_1 \text{ and } Cat_2 \text{ and } Cat_3 \text{ and } Cat_4) = P(Cat_1) * P(Cat_2) * P(Cat_3) * P(Cat_4)$$

Again, the probability for each cat is the same, .63, so our equation becomes:

$$P(Cat_1 \text{ and } Cat_2 \text{ and } Cat_3 \text{ and } Cat_4) = (.63)(.63)(.63)(.63)$$

Which can be simplified with some exponent rules to:

$$= (.63)^4$$

Which when plugged into a calculator will give us a final answer of:

$$= .158$$

Notice that this pattern can be expanded for any number of cats. So if we wanted to know the probability of finding 500 cats who lived past age twelve, we would calculate $(.63)^{500}$.

Now for the last part of the problem:

3. ***Find the probability that in a sample of 13 cats, AT LEAST one cat dies before age 12.***

This is an example of a problem where we can take advantage of the Complement Rule. Using it, we know that the probability that at least one cat dies is equal to 1 minus the probability that all 13 cats live. We can write this out as:

$$P(at\ least\ one\ dies) = 1 - P(all\ 13\ cats\ live)$$

Now, all we need to do is calculate the probability that all 13 cats will live past age twelve and plug this value into the formula. Luckily, we know from the earlier parts of the problem, that this probability will just be equal to $(.63)^{13}$.

We can plug this into the formula and solve to get:

$$P(at\ least\ one\ dies) = 1 - (.63)^{13}$$

$$= 1 - 0.0025$$

$$= 0.9975$$

So, we know that the probability of at least one cat dying before age 12 in a random sample of 13 cats is roughly 99.75%.

## Conditional Probability Rule ("GIVEN")

Finally, we have one more important rule: the Conditional Probability Rule. We use this rule when we can't assume that two events are independent. For example, if the two cats in the last problem had been siblings. The probability of one cat living longer might have been influenced by the other cat. Formally, the **Conditional Probability Rule** is written as:

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)}$$

The key phrase for problems that use this rule is the word "given." The probability of one event, A, happening <u>given</u> that another event, B, has already occurred.

Let's look at an example of a common problem that uses the conditional probability rule.

## Practice #3

*Given a sample of 258 people, the following table was created:*

|  | Male | Female | Totals |
|---|---|---|---|
| **Never Married** | 44 | 39 | **83** |
| **Married** | 67 | 67 | **134** |
| **Widowed** | 4 | 11 | **15** |
| **Divorced** | 11 | 15 | **26** |
| **Totals** | **126** | **132** | **258** |

1. ***Find the probability that a randomly selected individual was never married.***

For this first part, we won't need the conditional probability formula. We just need to set up a simple ratio:

$$P(Never\ Married) = \frac{Total\ Never\ Married}{Grand\ Total}$$

From the table, we can see that the total number of people who were never married is 83. The grand total is 258, the number of people in the sample. So we have:

$$P(Never\ Married) = \frac{83}{258} = 0.322$$

2. ***Find the probability that a randomly selected individual was never married, GIVEN that they are male.***

Now we are asked to find $P(Never\ Married\ |Male)$. Notice the keyword "given" in this question. This is a cue to use the conditional probability formula.

$$P(Never\ Married|Male) = \frac{P(Never\ Married\ \textbf{AND}\ Male)}{P(Male)}$$

So, we need to find two things: $P(Never\ Married\ \textbf{AND}\ Male)$ and $P(Male)$.

Let's start with $P(Male)$. We can find this the same way that we found $P(Never\ Married)$ in the first part of the question.

$$P(Male) = \frac{Total\ Male}{Grand\ Total} = \frac{126}{258} = 0.488$$

Next, let's find $P(Never\ Married\ AND\ Male)$. The ratio we need here is:

$$P(Never\ Married\ AND\ Male) = \frac{Number\ both\ Male\ and\ Never\ Married}{Grand\ Total}$$

The number of people who are both male and never married is found in the upper left box in the intersection of both categories, 44.

So, our equation becomes:

$$P(Never\ Married\ \boldsymbol{AND}\ Male) = \frac{Number\ both\ Male\ and\ Never\ Married}{Grand\ Total} = \frac{44}{258} = 0.171$$

Now for the final step, we plug our values into the conditional probability formula.

$$P(Never\ Married|Male) = \frac{P(Never\ Married\ \boldsymbol{AND}\ Male)}{P(Male)} = \frac{0.171}{0.488} = 0.349$$

So, we know that the probability of selecting an individual who has never married given that they are male is roughly **34.9%.**

And with that, we have covered three of the most common general probability concepts.

## Outro

Thank you for watching this TutorTube presentation! I hope you enjoyed this video. Please subscribe to our channel for more exciting videos. Check out the links in the description below for more information about The Learning Center and follow us on social media. See you next time!

## References

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