TutorTube: Indefinite Integration by Parts

Fall 2020

Introduction

Hello and welcome to TutorTube, where The Learning Center's Lead Tutors help you understand challenging course concepts with easy to understand videos. My name is Aima, Lead Tutor for MATH 1720. In today's video, we will explore integration by parts. Let's get started!

Integration

Before beginning integration by parts, you should be familiar with the term and the process of integration.
Integration is simply finding the antiderivative of a function.

Let's try a quick example before we continue:
Let us solve the following integral:

$$\int 3x^2 + 4x^3 + 2\,dx$$

For this integral, we have simpler terms which when integrated, would result in the form $\frac{x^{n+1}}{n+1} + C$. Where n is the respective power of each x.
So, we apply that to each of the terms, multiplying each of the coefficients across them as so:

$$\int 3x^2 + 4x^3 + 2\,dx = 3\left(\frac{x^{2+1}}{2+1}\right) + 4\left(\frac{x^{3+1}}{3+1}\right) + 2\left(\frac{x^{0+1}}{0+1}\right) + C$$

$$= 3\left(\frac{x^3}{3}\right) + 4\left(\frac{x^4}{4}\right) + 2\left(\frac{x^1}{1}\right) + C$$

We can simplify this by canceling out terms of multiplication we appropriate, and we would now have a cleaned-up function as:

$$= x^3 + x^4 + 2x + C$$

So we can finalize that $\int 3x^2 + 4x^3 + 2\,dx = x^3 + x^4 + 2x + C$

Now that we have attempted some integration examples, let's go ahead and begin integration by parts.

Integration by Parts

Integration by parts is the process of finding the antiderivative of a function, by breaking the function into three major parts. This splitting of the function is performed to enable us find a solution to the problem. We would have to introduce two new variables, u and v which are both functions of x, to be used in a special formula that is required to perform integration by parts:

$$\int udv = uv - \int vdu$$

Please take note that du is derived from u, while v is gotten by integrating dv.

To use this formula to find solutions to integration by parts problems, we consider a method of hierarchy that lets us know which parts of the function can be assigned to u first, before other parts can be assigned to dv. This is known as LIATE, where:

$$L = logarithmic$$

$$I = Inverse\ trig$$

$$A = Algebraic$$

$$T = Trigonometric$$

$$E = Exponential$$

Using the information we just covered, let us try an example. Let us find the integral of ln $(x)dx$.

$$\int \ln(x)\ dx$$

Because we cannot easily solve this type of problem, we would apply integration by parts.

We start by analyzing the function via the LIATE hierarchy.

- $\ln(x)$ is a logarithmic function
- dx is the remaining part of the problem

The $\ln(x)$ in the problem can be assigned to u, since it is the first logarithmic function we have. Then, we can assign everything else to dv, and it would look like:

$$u = ln(x) \qquad dv = dx$$

Now we solve for du and v:

$u = \ln(x)$

$du = \frac{1}{x}\ dx$

$dv = dx$

$v = \int dv = \int dx$

$v = x$

If we place u, v, du, and dv in the formula, we have:

$$\int \ln(x)\,dx = uv - \int v du \rightarrow \int \ln(x)\,dx = (\ln(x))(x) - \int (x)\left(\frac{1}{x}\right) dx$$

Now, we simplify the integral on the extreme right, and solve to get the solution:

$$\int \ln(x)\,dx = (\ln(x))(x) - \int x\left(\frac{1}{x} dx\right)$$
$$\int \ln(x)\,dx = (\ln(x))(x) - \int dx$$
$$\int \ln(x)\,dx = (\ln(x))(x) - x$$

Now, all we need to do is clean up the entire equation to get:

$$\int \ln(x)\,dx = xln(x) - x$$

Outro

Thank you for watching TutorTube! I hope you enjoyed this video. Please subscribe to our channel for more exciting videos. Check out the links in the description below for more information about The Learning Center and follow us on social media. See you next time!